# Stability Configurations of Electrons on a Sphere

By Michael Goldberg

**Abstract.** Stable arrangements of electrons in equilibrium on the surface of a sphere have been obtained for most of the cases up to 14 electrons. They resemble the arrangements of the contact points of the faces of an $n$-hedron of least volume circumscribed about the sphere, except for $n = 8$. In this case, it is shown that the electrons near the contact points are in equilibrium, but the arrangement is not stable.

In connection with his attempts to devise a model for the atom, J. J. Thomson proposed the problem of the determination of the stable configurations of a given number ($n$) of electrons bound to the surface of the sphere of unit radius and interacting under mutual repulsion. At each stable position, the potential energy is a minimum. This physical model of the atom has been abandoned, but the mathematical problem still remains unsolved, except for a few special cases. The results for $n = 5, 6, 7, 8, 10, 12$ and $14$ were determined by Föppl [1]. Cohn [2] summarized the results of Föppl and added the new special cases $n = 9$ and $n = 11$.

The author had published a paper on a similar problem [3], namely, the problem of minimizing $K$ for a polyhedron of a given number ($n$) of faces, where $K = F^3/V^2$ ($F$ = surface, $V$ = volume). For $n = 8$, he submitted the configuration designated by 8(2, 2, 2, 2). This indicates that the contact points of the faces lie, two by two, on four latitudes of a sphere. The distances of the contact points from the plane of the equator are $z_1 = .800$ and $z_2 = .265$. If the electrons are placed at these eight points, the potential energy is less than for the arrangement 8(1, 3, 3, 1), but more than for the arrangement 8(4, 4). As in Cohn's computation, the ordinates were then adjusted until the sum of the tangential forces on each electron was reduced to zero. Then, $z_1 = .785$ and $z_2 = .306$. The tabulated results for $n = 8$ are given in the following table.

| | *Potential problem* | | *Isoperimetric problem* | |
|---|---|---|---|---|
| *Arrangement* | *Parameters* | *Potential* $= \sum 1/d_{ij}$ | *Parameters* | $K = F^3/V^2$ |
| Vertices of cube 1, 3, 3, 1 (unstable) | | 39.45 | | 187.06 |
| 4, 4, (stable) | $z = .56$ | 39.32 | $z = .577$ | 181.55 |
| 2, 2, 2, 2, (unstable) | $z_1 = .785$<br>$z_2 = .306$ | 39.39 | $z_1 = .800$<br>$z_2 = .265$ | 180.93 |

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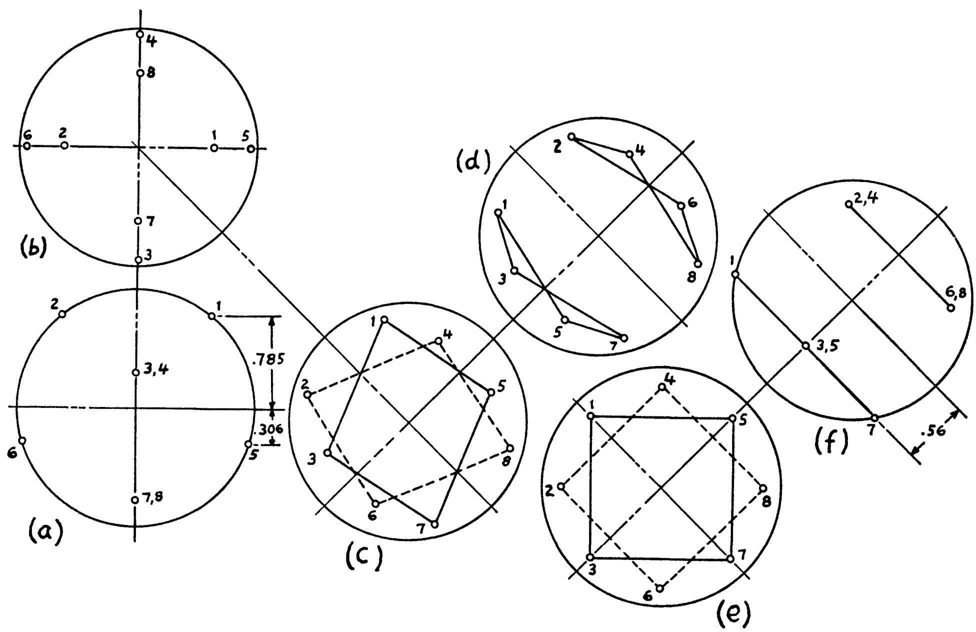
However, the arrangement 8(2, 2, 2, 2), shown in the views (a) and (b) of the figure, is unstable. It can be modified continuously, with a monotonically decreasing potential energy, to reach the arrangement 8(4, 4) described by Föppl, and shown in views (e) and (f). This can be seen by noting that the four points 1, 3, 7, 5, shown in views (c) and (d), make a skew quadrilateral whose sides are unequal and whose diagonals are unequal. This quadrilateral can be deformed continuously to reach the square array shown in view (e). Also, the configuration of the points 2, 4, 8, 6 can be similarly changed. These two sets of points can then undergo relative rotation about the axis through their centers to obtain a 45° displacement between the longitudes of the eight points.




*Eight Electrons on a Sphere*

For larger values of $n$, there may exist more than one stable arrangement of $n$ electrons on a sphere. Such multiplicities exist for the isoperimetric problem and for packing problems. All the stable arrangements found to date are the same as for the isoperimetric problem with a small change in the parameters, except for $n = 8$. For $n = 11$, the isoperimetric problem has not yet been solved.

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1. L. Föppl, "Stabile Anordnungen von Elektronen im Atom," *J. Reine Angew. Math.*, v. 141, 1912, pp. 251–301.
2. H. Cohn, "Stability configurations of electrons on a sphere," *MTAC.*, v. 10, 1956, pp. 117–120. MR 18, 356.
3. M. Goldberg, "The isoperimetric problem for polyhedra," *Tôhoku Math. J.*, v. 40, 1935, pp. 226–236.